Turbines

Wind Turbine Calculation

Betz Limit = 0.59 or 59%

$$\frac{1}{2}\rho A V^3 C_p$$

ρ = air density

v^3 = wind speed

C_p = power coefficient

L = blade length = $A = \pi r^2$

Example

Blade length, L = 52 m

Wind speed, v = 12 m/sec

Air density, ρ = 1.23 kg/m3

Power Coefficient, Cp = 0.4

Solution

Blade length

L = 52

r = 52m

$A = \pi r^2$

$\pi \times 52^2 = 8494.7$

$A = 8494.7\text{m}^2$

$$P_{available} = \frac{1}{2}\rho A V^3 C_p$$

$$\frac{1}{2} \times 1.23 \times 8494.7 \times 12^3 \times 0.4 = 3610995$$

$P_{available} = 3.61\text{MW}$

Marine Turbine Calculation

Betz Law Equivalent = 0.35 or 35%

$$\frac{p_w A V^3 C_{pm}}{2}$$

ρ_w = water density

v^3 = tidal flow speed

C_{pm} = power coefficient = 0.35max

L = blade length = $A = \pi r^2$

Example

Blade length, L = 26 m

Tidal flow speed, v = 4.4 m/sec (max)

Density of water, ρ = 1000 or 1025kg/m3

Power Coefficient, C_{pm} = 0.35

Solution

Blade length

L = 9

r = 9m

$A = \pi r^2$

$\pi \times 9^2 = 2123.7$

$A = 254.5\text{m}^2$

$$P_{available} = \frac{p_w A V^3 C_{pm}}{2}$$

$$\frac{1000 \times 254.5 \times 4.4^3 \times 0.35}{2} = 3793882.4$$

$P_{available} = 3.8\text{MW}$

Find Velocity Wind Turbine

$$\frac{1}{2} \text{x } 1.23 \text{ x } 8494.7 \text{ x } ? \text{ x } 0.4 = 3610995$$

$$3610995 \div \left(\frac{1}{2} \text{ x } 1.23 \text{ x } 8494.7 \text{ x } 0.4\right)$$

$$\text{Answer} = 1728$$

$$\sqrt[3]{1728} = 12$$

$$\text{v} = 12$$

$$\text{v} = 12^3$$

$$\mathbf{P_{available} = \frac{1}{2}\rho AV^3C_p}$$

$$\frac{1}{2} \text{x } 1.23 \text{ x } 8494.7 \text{ x } 12^3 \text{x } 0.4 = 3610995$$

$$\text{P}_{\text{available}} = 3.61\text{MW}$$

Find Velocity Marine Turbine

$$\frac{1}{2} \text{x } 1000 \text{ x } 254.5 \text{ x } ? \text{ x } 0.35 = 3793882.4$$

$$3793882.4 \div \left(\frac{1}{2} \text{ x } 1000 \text{ x } 254.5 \text{ x } 0.35\right)$$

$$\text{Answer} = 85.184$$

$$\sqrt[3]{85.184} = 4.4$$

$$\text{v} = 4.4$$

$$\text{v} = 4.4^3$$

$$\mathbf{P_{available} = \frac{p_w AV^3 C_{pm}}{2}}$$

$$\frac{1000 \text{ x } 254.5 \text{ x } 4.4^3 \text{x } 0.35}{2} = 3793882.4$$

$$\text{P}_{\text{available}} = 3.8\text{MW}$$

Wind Turbine Farm Spacing factor = 3 rotor diameters apart, 8 rotors behind.

9 x turbines = 32.49MW

$\mathbf{T_1}$ $\mathbf{T_2}$ $\mathbf{T_3}$
$\mathbf{T_4}$ $\mathbf{T_5}$ $\mathbf{T_6}$
$\mathbf{T_7}$ $\mathbf{T_8}$ $\mathbf{T_9}$

$$\frac{3.61\text{x}10^6}{(3 \text{ x } 8 \text{ x } 104)^2} = 0.58 \text{ W/m}^2$$

Check Answer

$$\text{Area} = (8 \ x \ 3 \ x \ 104)^2 = 6230016$$

$$6230016 \text{ x } 0.58 = 3.61\text{MW}$$

$$3.61 \text{ x } 9 = 32.49\text{MW}$$

$$\mathbf{0.58 \text{ watts per meter squared}}$$

Marine Turbine

Water is 800 times denser than air so tidal energy is far more powerful than that of wind energy. Unlike wind, tides are predictable and stable; therefore, tidal generators are steady and produce a reliable stream of electricity. The downside is that they are far more expensive due to weight and ingress protection. Maintenance is also much harder to complete and comes with serious risks to health.

Blade Tip Speed Ratio

$\text{Turbine Diameter} = 104\text{m}$

$\text{RPM} = 15$

$\text{Wind speed, v} = 12\ \text{m/sec}$

$$\text{Blade tip speed } = \frac{\text{RPM x } \pi \text{D}}{60} = \frac{15 \text{ x } \pi \text{ x } 104}{60} = 81.7\text{m/sec}$$

$$\lambda = \frac{\text{Blade tip speed}}{\text{Wind speed}} = \frac{81.7}{12} = 6.8 \text{ (High Performance)}$$

$$\textbf{Efficiency} = \frac{\mathbf{p_{out}}}{\mathbf{p_{in}}}$$

$$p_{in} = \frac{1}{2} \text{x } 1.23 \text{ x } (\pi \text{ x } 52^2) \text{ x} 12^3 = 9027664.564\text{W}$$

$$p_{out} = 0.4 \text{ x } 9027664.564 = 3.61\text{MW}$$

$$\text{Efficiency} = \frac{3.61 \text{x} 10^6}{9027664.564} = 0.4 = 40\%$$

Energy Per Year (MGW/h)

$\text{Energy} = \text{Power x time}$

$3.61 \text{ x } 24 \text{ x } 365 = 31623.6\text{MGW/h}$

Volume Of Air

Area x Volume x Time

$\text{Avt} = \frac{1}{2}\text{mv}^2 = \frac{1}{2}\rho\text{Avtv}^2 = \frac{1}{2}\rho\text{Atv}^3$

Note that power = energy/time

Q1) Identify five factors which might reduce the overall operational efficiency of wind turbines, and for each factor briefly explain how these limitations could be reduced.

Location of Turbines

If the wind turbines are situated close to build up areas or the bottom of valleys this will reduce efficiency due to limited wind. To add to location implementation Wind turbines produce a lot of machine noise and therefore considered a noisy eyesore. For optimum performance and to counter community protests, wind turbine should be located offshore, coastal areas, hill tops and discreet open planes where they can do their job without causing offence.

Positioning Turbines

Wind turbines extract energy from the wind. If the wind turbine arrays are positioned to close to each other they cause wind shadows (wakes). A wind farm turbine layout must consider that turbines close together produce wakes which then interfere with turbine arrays depending on the wind direction and spacing factor. The consequences of wakes (shadowing) reduces wind speed. The wake effect is the aggregated influence on the energy production of a wind farm, the changes in wind speed reduces the overall efficiency of the turbine farm. In some instances with low distance separation, wind farm owners have to sacrifice select turbines to produce optimum efficiency due to poor planning, lack of space or wind direction. To combat wakes the spacing distance of turbines needs to be correct. The rule of thumb for spacing is, three times the rotor diameter abreast, and eight times the rotor diameter downwind, this method is proven to maximise performance of turbine arrays.

Wind speed

The wind speed directly effects a wind turbines performance, although some wind turbines can operate at just 0.5mph the normal wind speed to produce significant power is around 6-7mph, at these wind speeds the energy harvested will be low. Wind speeds higher than 50-55mph exceed the upper safety limit (survival speed) and the turbine has to be shut down. The optimum wind speed is around 30mph. Therefore, location is very important.

Air Density

The kinetic energy of a moving body is proportional to its mass The kinetic energy in the wind depends on the density of the air. Low air density effects a wind turbines performance, avoiding mountainous regions is advised as low air pressure and density is much lower than that of sea level. Humidity also lowers air density. Selecting areas with high air density increase overall efficiency.

Blade Type and Radius

The number of blades is determined by a variety of factors including cost versus performance, noise and visual appearance. Three blades are preferred as they perform better in low speeds. By using smaller rotor blades when a bigger rotor blade (swept area) could be implemented. The blade length directly relates to the energy harvested.

Hydropower

$$P = mgH_{net}\,\eta$$

P = power, measured in Watts (W).

m = mass flow rate in kg/s (numerically the same as the flow rate in litres/second because 1 litre of water weighs 1 kg)

g = the gravitational constant, which is $9.81 m/s^2$

H_{net} = the net head. This is the gross head physically measured at the site, less any head losses. To keep things simple head losses can be assumed to be 10%, so $H_{net} = H_{gross} \times 0.9$

η = the product of all of the component efficiencies, which are normally the turbine, drive system and generator

For a typical small hydro system, the turbine efficiency would be 85%, drive efficiency 95% and generator efficiency 93%, so the overall system efficiency would be:

η $= 0.85 \times 0.95 \times 0.93 = 0.751$ i.e. 75.1%

Therefore, if you had a relatively low gross head of 2.5 metres, and a turbine that could take a maximum flow rate of 3 m^3/s, the maximum power output of the system would be:

First convert the gross head into the net head by multiplying it by 0.9, so:

$\mathbf{H_{net}} = H_{gross} \times 0.9 = 2.5 \times 0.9 = 2.25$ m

Then convert the flow rate in m^3/s into litres/second by multiplying it by 1000, so:
3 m^3/s = 3,000 litres per second

Power (W) $= m \times g \times H_{net} \times \eta = 3{,}000 \times 9.81 \times 2.25 \times 0.751 = 49{,}729$ W = 49.7 kW

Flow rate through the turbine is 150 litres / second.

In this case $H_{net} = 50 \times 0.9 = 45$ m and the flow rate in litres/second is 150, hence:

Power (W) $= m \times g \times H_{net} \times \eta = 150 \times 9.81 \times 45 \times 0.751 = 49{,}729$ W = 49.7 kW